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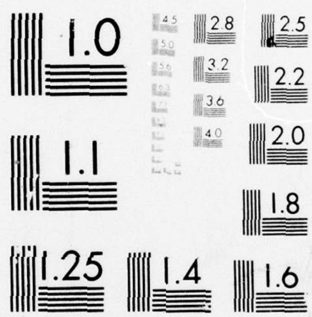

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9 TECHNICAL MEMORANDUM 3-77

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STOCHASTIC DUELS WITH DISPLACEMENTS (SUPPRESSION).

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AD No.
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15 DAHC04-72-A-0001

DEPARTMENT OF THE ARMY
US ARMY TRADOC SYSTEMS ANALYSIS ACTIVITY
WHITE SANDS MISSILE RANGE
NEW MEXICO 88002

391 859 JOB

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)

TECHNICAL MEMORANDUM 3-77, "STOCHASTIC DUELS WITH DISPLACEMENTS (SUPPRESSION)

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER
USA TRASANA

7. AUTHOR(s)

G. TREVOR WILLIAMS
University of Bristol, Bristol, England

8. CONTRACT OR GRANT NUMBER(s)

DAHC04-72-A-0001
Task ~~Oder~~ 76-10

9. PERFORMING ORGANIZATION NAME AND ADDRESS

US Army TRADOC Systems Analysis Activity
ATTN: ATAA-TG
White Sands Missile Range, NM 88002

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS

Commander, US Army Research Office
ATTN: DRXRO-PR
P.O. Box 12211, Research Triangle Pk, NC 27709

12. REPORT DATE
March 1978

13. NUMBER OF PAGES
9

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

Battelle Columbus Laboratories
P.O. Box 8832
Durham, NC 27707

15. SECURITY CLASS. (of this report)

UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release - distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The stochastic duel is extended to include the possibility of a near-miss on each round fired, which causes the opponent to displace. During displacement, the displacing contestant cannot return the fire but is still a target for his opponent. An alternative interpretation of this model is to consider the displacement time as the time a contestant's fire is suppressed by his opponent's fire and that he does not move but merely ceases fire temporarily. All times are exponentially distributed.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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US ARMY TRADOC SYSTEMS ANALYSIS ACTIVITY
WHITE SANDS MISSILE RANGE
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TRASANA MEMORANDUM 3-77

STOCHASTIC DUELS WITH DISPLACEMENTS (SUPPRESSION)[1]

BY

G. TREVOR WILLIAMS[2]

ABSTRACT

The stochastic duel is extended to include the possibility of a near-miss on each round fired, which causes the opponent to displace. During displacement, the displacing contestant cannot return the fire but is still a target for his opponent. An alternative interpretation of this model is to consider the displacement time as the time a contestant's fire is suppressed by his opponent's fire and that he does not move but merely ceases fire temporarily. All times are exponentially distributed.

---

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Some earlier work has considered mobility by incorporating varying projectile time of flight, [1] and [2], or by varying hit probability, [3], [4], and [5]. Some very simple models where the contestants displace upon receiving a near-miss are provided in [6], [7], and [8]. In this paper, we consider a more general situation of the latter type.

NOTE: The numbers in brackets pertain to the references listed at the end of the paper.

In general, most earlier studies of stochastic duels have not considered an interaction between the two contestants, A and B. That is to say, the same results would have been effected if each fired separately at his own target in two different localities and they subsequently compared their times to score a kill, having previously agreed that the quicker of the two would be the winner. We shall now consider a duel where A's behavior is contingent upon B's, and vice versa.

We assume, as usual, that both duelists begin to load and fire simultaneously, but we now add the possibility that one or the other may score a near-miss, the effect of which on a duelist is to make him move to a new firing position. We may imagine that if he did not move, he would immediately be killed since his opponent has now gotten his range. During his displacement time, he is subject to fire from his opponent but cannot return it. We assume that the probabilities of a near-miss and a kill are the same from round to round and that once a duelist has displaced because of a near-miss, he proceeds to load and fire as before. His displacement and firing times are random variables whose probability density functions are known and are not necessarily the same.

An alternate interpretation that may be useful is as follows: Upon receiving a near-miss, the duelist merely seeks cover in his present position and ceases fire for a period of time equal to the corresponding displacement time, i.e., merely interpret displacement time as fire-suppression time, and the model is a fire-suppression model as it stands.

Our first step is to eliminate from conside-ation all complete misses on both sides. We are thus left with a series of near-misses that form, so to speak, a succession of turning points on which the duel hinges. It does not matter which of the duelists scores a near-miss since it still interrupts the course of the duel. We thus make a list of when the near-misses occurred and who scored them. Ultimately, the duel ends on a near-miss which was actually a kill.

We introduce the following notation. Let

$$\left.\begin{array}{l} p_{AA} = \text{the conditional probability that A scores the next near-miss given that A scored the last one} \\ p_{AB} = \text{the conditional probability that B scores the next near-miss given that A scored the last one} \end{array}\right\} \quad (1)$$

and similarly for p_{BA} and p_{BB}. Then, we plainly have

$$p_{AA} + p_{AB} = p_{BA} + p_{BB} = 1 \; . \qquad (2)$$

Next, let

p_A = the unconditional probability that A scores a near-miss on any round (3)

k_A = the conditional probability that A scores a kill given a near-miss (4)

that is, on any near-miss, a kill may or may not have occurred, and

$k_A p_A$ = the joint probability of a near-miss and a kill or just simply a kill (5)

Also, let

$q_A = 1 - p_A$ = the probability that A misses entirely on any round (6)

and

$l_A = 1 - k_A$ = the conditional probability that A does not score a kill given a near-miss (7)

Of course, there is an identical set of notation for B. We further let

p_{OA} = the probability that A scores the first near-miss

p_{OB} = the probability that B scores the first near-miss (8)

so that

$$p_{OA} + p_{OB} = 1 \qquad (9)$$

and we call

$$\left.\begin{aligned} P_n(A) &= \text{the probability that A kills, for the first time, on near-miss number n} \\ P_n(B) &= \text{the probability that B kills, for the first time, on near-miss number n} \end{aligned}\right\} \qquad (10)$$

Then, the probability, $P(A)$, that A wins the duel is

$$P(A) = \sum_{n=1}^{\infty} P_n(A)$$

and similarly for B

$$P(B) = \sum_{n=1}^{\infty} P_n(B) \qquad (11)$$

also, $P(A) + P(B) = 1$.

Clearly, when $n = 1$, we have

$$P_1(A) = p_{0A} k_A \text{ and } P_1(B) = p_{0B} k_B\ . \qquad (12)$$

Now, the probability that A scored a near-miss on round n-1 but did not kill is given by $\frac{l_A}{k_A} P_{n-1}(A)$ since $P_{n-1}(A)$ includes a factor k_A for the kill on round n-1, which factor must be replaced by l_A if there was no kill. Thus,

$$P_n(A) = k_A \left[p_{AA} \frac{l_A}{k_A} P_{n-1}(A) + p_{BA} \frac{l_B}{k_B} P_{n-1}(B) \right] \qquad (13)$$

provides the two near-miss situations on a given round that can lead to a kill by A on the next round. Hence,

$$P_n(A) = l_A\, p_{AA}\, P_{n-1}(A) + l_B\, p_{BA} \frac{l_A}{k_B} P_{n-1}(B) \qquad (14)$$

and similarly,

$$P_n(B) = l_B\, p_{BB}\, P_{n-1}(B) + l_A\, p_{AB}\, \frac{k_B}{k_A}\, P_{n-1}(A)\,. \tag{15}$$

Summing both sides of these simultaneous difference equations from 2 to ∞ and making use of Equations (11) and (12), we find that

$$P(A) - p_{0A}k_A = l_A\, p_{AA}\, P(A) + l_B\, p_{BA}\, \frac{k_A}{k_B}\, P(B) \tag{16}$$

and

$$P(B) - p_{0B}\, k_B = l_B\, p_{BB}\, P(B) + l_A\, p_{AB}\, \frac{k_B}{k_A}\, P(A). \tag{17}$$

When we solve these two equations simultaneously for P(A), we obtain

$$P(A) = \frac{k_A\, (p_{0A}\, k_B + l_B\, p_{BA})}{k_B - l_A\, p_{AA}\, k_B + l_B\, p_{BA}\, k_A} \tag{18}$$

with a corresponding expression for P(B). By using Notations (1), (2), and (7), it is easily shown that the denominators of P(A) and P(B) are equal.

Now, adding the numerator of P(A) to that of P(B), we find

$$k_A\, k_B\, p_{0A} + k_A\, l_B\, p_{BA} + k_A\, k_B\, p_{0B} + k_B\, l_A\, p_{AB}$$

which, by using Equations (2) and (9), is equal to

$$k_A\, k_B + k_A\, l_B\, p_{BA} + k_B\, l_A - k_B\, l_A\, p_{AA}\,.$$

And now, using Equation (7), this is equal to

$$k_B + k_A\, l_B\, p_{BA} - k_B\, l_A\, p_{AA}$$

which is equal to the common denominator of P(A) and P(B), thus checking that $P(A) + P(B) = 1$.

Now, assume that A's firing time is exponentially distributed with mean $1/r_A$ (r_A is A's rate of fire), and his displacement time is also an exponential with mean δ_A, and similarly for B. Then, by exactly the same reasoning as used in the fundamental duel, Reference [9], to arrive at the first kill, the probability that A makes the first near-miss, p_{0A}, is

$$p_{0A} = \frac{p_A r_A}{p_A r_A + p_B r_B} \quad . \tag{19}$$

Also, from Notation (1),

$$p_{AA} = P[T_A \leq T_B + D_B] \tag{20}$$

where

T_A = random variable time between A's near-misses

T_B = time between B's near-misses

D_B = B's time to displace .

This expression accounts for the fact that after A scores a near-miss, he proceeds to fire again, whereas B must first displace and then fire. And, as we have already seen, when the probability of the event (near-miss) is P_A and the firing time is *exponential with mean* $1/r_A$, *then the time to* score a near-miss is exponentially distributed with mean $1/p_A r_A$. Again, we may use the of the fundamental duel to obtain this probability (p_{AA}) by noting that the characteristic function of $T_B + D_B$ is simply the product of their individual chataceristic functions. Hence,

$$p_{AA} = \frac{1}{2\pi i} \int_L \frac{p_A r_A \, p_B r_B \, du}{(p_A r_A + iu)(p_B r_B - iu)(1 - i\delta_B u)u} \tag{21}$$

and using residue theory, we have

$$p_{AA} = \frac{p_A r_A \,(1 + p_A r_A \,\delta_B + p_B r_B \,\delta_B)}{(p_A r_A + p_B r_B)\,(1 + p_A r_A \,\delta_B)} \tag{22}$$

and similarly,

$$p_{BA} = \frac{p_A r_A}{(p_A r_A + p_B r_B)\ (1 + p_B r_B\ \delta_A)} \tag{23}$$

When we substitute these results into Equation (18), we obtain after a little algebra

$$P(A) = \frac{k_A p_A r_A (1 + p_A r_A\ \delta_B)(1 + k_B p_B r_B\ \delta_A)}{k_A p_A r_A (1 + p_A r_A\ \delta_B)(1 + k_B p_B r_B\ \delta_A) + k_B p_B r_B (1 + p_B r_B\ \delta_A)(1 + k_A p_A r_A\ \delta_B)} \cdot \tag{24}$$

It is interesting to note that if $k_A = k_B = 1$, then all near-misses are really kills, and Equation (24) reduces to the solution for the fundamental duel with exponential firing times, Reference [9], and the same result occurs if $\delta_A = \delta_B = 0$ (zero displacement), as it should.

The expression in Equation (24) is unwieldy since it contains eight parameters. If we define

$$P_B = p_B r_B\ \delta_B$$

$$P_A = p_A r_A\ \delta_A$$

$$D = \delta_B / \delta_A$$

then Equation (24) becomes

$$P(A) = \frac{D\ k_A\ P_A\ (D + k_B\ P_B)(1 + D\ P_A)}{k_A\ D\ P_A (D + k_B\ P_B)(1 + D\ P_A) + k_B\ P_B (D + P_B)(1 + k_A\ D\ P_A)} \tag{25}$$

which reduces the parameters to only five.

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